



MATHEMATICS METHODS Year 12

Section One: Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1

(4 marks)

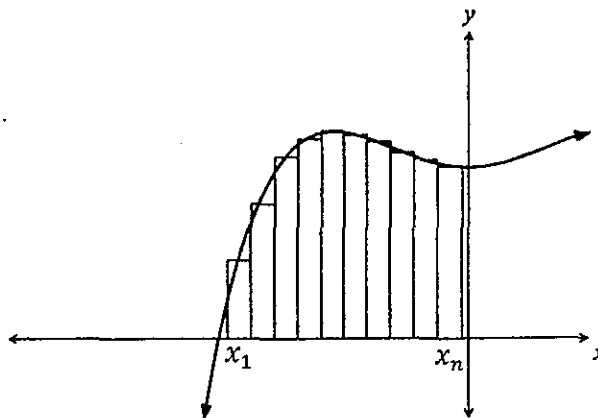
At any point (x, y) on a particular curve $\frac{d^2y}{dx^2}$ is a quadratic function of x .

(a) From the information about $\frac{d^2y}{dx^2}$,

(i) what is the maximum number of stationary points for this original curve? (1 mark)

(ii) what is the maximum number of points of inflection for this original curve? (1 mark)

(b) Part of the curve is shown below. The rectangles can be used to approximate the area under the curve.



(i) What is represented by the expression $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i) \times \Delta x \right)$? (1 mark)

(ii) Simplify the expression in (i) above, using Calculus symbols. (1 mark)

Question 2**(4 marks)**

At any point (x, y) , a particular curve is defined by $\frac{d^2y}{dx^2} = 1 - x^2$.

A tangent drawn to the curve at $(1,1)$ has equation $y = 2 - x$.

Determine the equation of the curve.



Question 3

(4 marks)

Differentiate each of the following with respect to x .

(a) $y = \frac{4}{x} - 3\sqrt{x^3}$ (Leave your answer with positive indices) (2 marks)

(b) $y = \frac{5-x}{5x+1}$ (Simplify your answer) (2 marks)

Question 4

(3 marks)

$g(x)$ is a function such that $g(-1) = 4$ and $g'(-1) = 2$.

$f(x)$ is a function such that $f(-1) = f'(-1) = 3$.

Determine $R'(-1)$ where $R(x) = f(x) \times (g(x))^2$

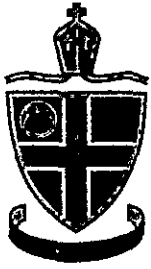
Additional working space

Question number: _____

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Christ Church
Grammar School

2019
TEST 5

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 5**(7 marks)**

Consider the graph of $f(x) = x(x - 1)(x + 2)$.

- (a) Determine, correct to two decimal places where necessary, the coordinates of:
- (i) the x -intercepts of $f(x)$. (1 mark)

 - (ii) the point(s) of inflection of $f(x)$. (1 mark)
- (b) Determine, the x -value(s), correct to two decimal places where necessary, where:
- (i) $f(x) > 0$. (2 marks)

 - (ii) $f'(x) < 0$. (2 marks)

 - (iii) $f''(x) > 0$. (1 mark)

Question 6

(6 marks)

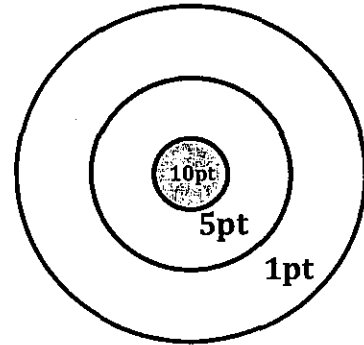
At Andre's gun shooting gallery a target is made up of three concentric circular regions. Andre is very accurate. When he shoots he never records a miss. Points are scored for hitting various parts of the target.

The innermost region (Bullseye) scores 10 points. The next outer region scores 5 points and the outer region scores 1 point.

The random variable X represents the number of points Andre scores.

Part of the probability distribution for X is given below.

x	1	5	10
$P(X = x)$	0.6	0.3	



- (a) Calculate $P(X = 10)$. (1 mark)

Andre fires two bullets at the target. Assume that each shot is independent of the other.

- (b) Calculate the probabilities of the following events:

- (i) The first bullet scores 5 and the second scores 1. (1 mark)

- (ii) The first bullet scores 5 or the second scores 1. (2 marks)

- (iii) The second bullet scores 5 given the first scored 1. (1 mark)

- (iv) Andre scores a total of 6 points. (1 mark)

Question 7

(9 marks)

The number of snow leopards in Siberia has been decreasing at a rate proportional to the number present from 1993. At the beginning of 1993 there were 440 snow leopards in Siberia. That is $\frac{dw}{dt} = -kw$ where k is the constant of proportionality and t is the number of years since 1993.

(a) Show clearly that $W = W_0 e^{-kt}$ satisfies the above equation. (2 marks)

By the beginning of 2000 there were only 356 snow leopards in Siberia.

(b) Determine: (1 mark)

(i) the value of W_0 .

(ii) the value of the constant of proportionality, correct to three decimal places. (2 marks)

Hence, or otherwise,

(c) determine the expected number of snow leopards at the beginning of 2010. (2 marks)

(d) determine during which year the number of snow leopards will first fall below 300. (2 marks)

Question 8**(4 marks)**

Let the proportion of parents at CCGS that support their son having one hour per night on electronic devices be \hat{p} . A random sample of n parents (where $n \geq 100$) was selected and 56 indicated that they supported the proposal. Find n if the magnitude of the margin of error for the 99% confidence interval for \hat{p} is 0.1.

Question 9**(4 marks)**

Let the proportion of students at CCGS who are left footed be π . A sample of 400 students at CCGS yielded a confidence interval for π as $0.23 \leq \pi \leq 0.29$.

(a) How many in this sample are left footed?

(1 mark)

(b) If 50 samples of 400 students each were selected, and the associated confidence intervals for π calculated in the same manner, how many of these confidence intervals would actually contain π ?

(3 marks)

Additional working space

Question number: _____





MATHEMATICS METHODS Year 12

Section One: Calculator-free

Your name MARK - KING. (1)

Teacher's name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

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Question 1

(4 marks)

At any point (x, y) on a particular curve $\frac{d^2y}{dx^2}$ is a quadratic function of x .

(a) From the information about $\frac{d^2y}{dx^2}$,

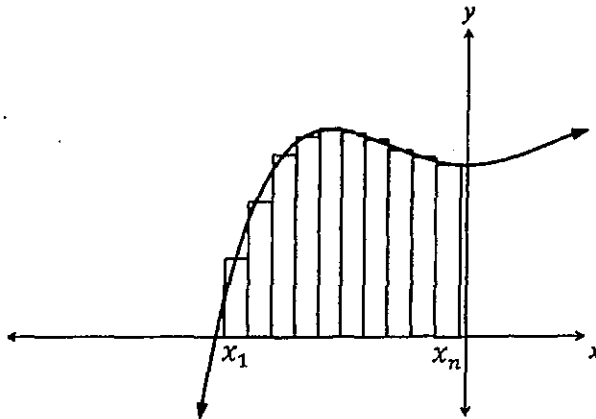
(i) what is the maximum number of stationary points for this original curve? (1 mark)

3 (original quartic) ✓

(ii) what is the maximum number of points of inflection for this original curve? (1 mark)

2 (original quartic) ✓

(b) Part of the curve is shown below. The rectangles can be used to approximate the area under the curve.



(i) What is represented by the expression $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i) \times \Delta x \right)$? (1 mark)

The estimated area under the curve ✓

(ii) Simplify the expression in (i) above, using Calculus symbols. (1 mark)

$\int_{x_1}^{x_n} f(x) dx$ (must have x_n and x_1) ✓

Question 2

(4 marks)

At any point (x, y) , a particular curve is defined by $\frac{d^2y}{dx^2} = 1 - x^2$.
 A tangent drawn to the curve at $(1,1)$ has equation $y = 2 - x$.

Determine the equation of the curve.

$$\frac{dy}{dx} = \underline{x - \frac{x^3}{3} + C_1} \quad \checkmark \quad (1^{st} \text{ derivative } + c)$$

$$\frac{dy}{dx} = -1 \quad \text{when } x=1 \quad \Rightarrow \quad \underline{C_1 = -1\frac{2}{3}} \quad \checkmark \quad (C_1 \text{ value})$$

$$\therefore y = \frac{x^2}{2} - \frac{x^4}{12} - \frac{5x}{3} + C_2$$

$$\text{Sub in } (1,1) \quad \therefore 1 = \frac{1^2}{2} - \frac{1^4}{12} - \frac{5(1)}{3} + C_2$$

$$\therefore \underline{C_2 = \frac{27}{12}} \quad \checkmark \quad (C_2 \text{ value})$$

$$\underline{y = \frac{x^2}{2} - \frac{x^4}{12} - \frac{5x}{3} + \frac{27}{12}} \quad \checkmark \quad (\text{equation})$$

Question 3

(4 marks)

Differentiate each of the following with respect to x .

(a) $y = \frac{4}{x} - 3\sqrt{x^3}$

(Leave your answer with positive indices)

(2 marks)

$$y = 4x^{-1} - 3x^{3/2}$$

$$\frac{dy}{dx} = -4x^{-2} - 3 \times \frac{3}{2} x^{1/2} \quad \checkmark \quad \left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{-\frac{4}{x^2} - \frac{9\sqrt{x}}{2}}{\quad} \quad \checkmark \quad \left(\frac{dy}{dx} + \text{positive indices}\right)$$

(b) $y = \frac{5-x}{5x+1}$

$$\frac{u'v - uv'}{v^2} \quad (\text{Simplify your answer})$$

(2 marks)

$$\frac{dy}{dx} = \frac{-1(5x+1) - (5-x)(5)}{(5x+1)^2} \quad \checkmark \quad \left(\frac{dy}{dx}\right)$$

$$= \frac{-1 + 5x - 25 + 5x}{(5x+1)^2}$$

$$= \frac{-26}{(5x+1)^2} \quad \checkmark \quad (\text{simplified})$$

Question 4

(3 marks)

$g(x)$ is a function such that $g(-1) = 4$ and $g'(-1) = 2$.

$f(x)$ is a function such that $f(-1) = f'(-1) = 3$.

Determine $R'(-1)$ where $R(x) = f(x) \times (g(x))^2$

$$u'v + uv'$$

$$R'(x) = \frac{f'(x) \cdot (g(x))^2 + f(x) \cdot 2g(x) \cdot g'(x)}{\quad} \quad \checkmark \text{ (Product Rule)}$$

$$= 3 \times 4^2 + 3 \times 2 \times (4) (2)$$

$$= \underline{48} + \underline{48} \quad \checkmark \text{ (substitution)}$$

$$= \underline{\underline{96}} \quad \checkmark \text{ (value)}$$



MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Your name MARK-KING (2)

Teacher's name _____

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Question 5

(7 marks)

Consider the graph of $f(x) = x(x-1)(x+2)$.

(a) Determine, correct to two decimal places where necessary, the coordinates of:

(i) the x -intercepts of $f(x)$. (1 mark)

$$\underline{(-2, 0) \quad (0, 0) \quad (1, 0)} \quad \checkmark \quad \text{(MUST HAVE ALL)}$$

(ii) the point(s) of inflection of $f(x)$. (1 mark)

$$\underline{(-0.33, 0.74)} \quad \checkmark \quad \text{(MUST BE TO 2DP)}$$

(b) Determine, the x -value(s), correct to two decimal places where necessary, where:(i) $f(x) > 0$. (2 marks)

$$\underline{-2 < x < 0} \quad \checkmark \quad \text{OR} \quad \underline{x > 1} \quad \checkmark$$

(ii) $f'(x) < 0$. (2 marks)

$$\underline{-1.22} \quad \checkmark < x < \underline{0.55} \quad \checkmark$$

(iii) $f''(x) > 0$. (1 mark)

$$\underline{x > -0.33} \quad \checkmark$$

Question 6

(6 marks)

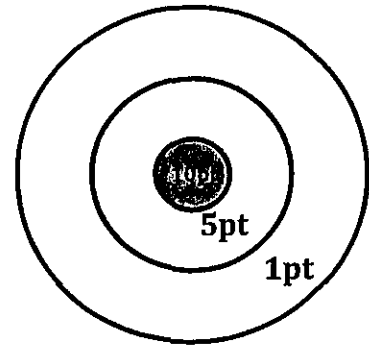
At Andre's gun shooting gallery a target is made up of three concentric circular regions. Andre is very accurate when he shoots he never records a miss. Points are scored for hitting various parts of the target.

The innermost region (Bullseye) scores 10 points. The next outer region scores 5 points and the outer region scores 1 point.

The random variable X represents the number of points Andre scores.

Part of the probability distribution for X is given below.

x	1	5	10
$P(X = x)$	0.6	0.3	0.1



- (a) Calculate $P(X = 10)$. (1 mark)

$$P(X=10) = 0.1 \checkmark$$

Andre fires two bullets at the target. Assume that each shot is independent of the other.

- (b) Calculate the probabilities of the following events:

- (i) The first bullet scores 5 and the second scores 1. (1 mark)

$$0.3 \times 0.6 = 0.18 \checkmark$$

- (ii) The first bullet scores 5 or the second scores 1. (2 marks)

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \checkmark \\ &= 0.72 \checkmark \end{aligned}$$

- (iii) The second bullet scores 5 given the first scored 1. (1 mark)

$$\frac{0.18}{0.6} = 0.3 \checkmark$$

- (iv) Andre scores a total of 6 points. (1 mark)

$$5 \neq 1 \quad \text{or} \quad 1 \neq 5$$

$$2 \times 0.18 = 0.36 \checkmark$$

6

Question 7

(9 marks)

The number of snow leopards in Siberia has been decreasing at a rate proportional to the number present from 1993. At the beginning of 1993 there were 440 snow leopards in Siberia. That is $\frac{dw}{dt} = -kw$ where k is the constant of proportionality and t is the number of years since 1993.

- (a) Show clearly that $W = W_0e^{-kt}$ satisfies the above equation. (2 marks)

$$\frac{dW}{dt} = (-k) W_0 e^{-kt} \quad \checkmark \quad (\text{differentiates})$$

$$\frac{dW}{dt} = -k W \quad \checkmark \quad (\text{Sub } W \text{ for } W_0 e^{-kt})$$

By the beginning of 2000 there were only 356 snow leopards in Siberia.

- (b) Determine:

(1 mark)

- (i) the value of W_0 .

$$W_0 = \underline{440} \quad \checkmark \quad (\text{Given})$$

- (ii) the value of the constant of proportionality, correct to three decimal places.

(2 marks)

$$356 = 440 e^{-7k} \quad \checkmark \quad (\text{Equation})$$

$$k = \underline{0.030} \quad \checkmark \quad (k \text{ to } 3dp)$$

Hence, or otherwise,

- (c) determine the expected number of snow leopards at the beginning of 2010.

$$W = 440 e^{-0.030(17)} \quad \checkmark \quad (\text{equation}) \quad (2 \text{ marks})$$

$$W \approx \underline{264} \text{ snow leopards} \quad \checkmark \quad (\text{Accept } \underline{263})$$

- (d) determine during which year the number of snow leopards will first fall below 300.

(2 marks)

$$300 = 440 e^{-0.03(t)}$$

$$t = 12.8$$

\checkmark
(t-value)

∴ During 2005 \checkmark
(Yr 2005)

See next page

9

Question 8

(4 marks)

Let the proportion of parents at CCGS that support their son having one hour per night on electronic devices be \hat{p} . A random sample of n parents (where $n \geq 100$) was selected and 56 indicated that they supported the proposal. Find n if the magnitude of the margin of error for the 99% confidence interval for \hat{p} is 0.1.

Point Estimate $\hat{p} = \frac{56}{n}$ ✓ (pt est.)

Since $n \geq 30$ (by CLT) $\hat{p} \Rightarrow$ Normal Dist.

Margin of error for 99% CI

$$0.1 = 2.576 \times \sqrt{\frac{\frac{56}{n} (1 - \frac{56}{n})}{n}} \quad \checkmark \quad \left(\begin{array}{l} \text{Equation with} \\ \text{ME} \end{array} \right)$$

Using Casd $n = -216.29, 62.6, 153.69$

Since $n \geq 100$

$n = \underline{153}$ ✓ (solⁿ 153)

Accept 154

$152 \Rightarrow 0.1007882969$
$153 \Rightarrow 0.1003202471$
$154 \Rightarrow 0.09985546279$

Question 9

(4 marks)

Let the proportion of students at CCGS who are left footed be π . A sample of 400 students at CCGS yielded a confidence interval for π as $0.23 \leq \pi \leq 0.29$.

(a) How many in this sample are left footed?

(1 mark)

$$\text{Point Estimate for } \pi, \hat{\pi} = \frac{0.23 + 0.29}{2}$$

$$\hat{\pi} = \underline{\underline{0.26}}$$

$$\therefore \text{N}^{\circ} \text{ left footers} = 400 \times 0.26 = \underline{\underline{104}} \checkmark \text{ (student N}^{\circ}\text{)}$$

(b) If 50 samples of 400 students each were selected, and the associated confidence intervals for π calculated in the same manner, how many of these confidence intervals would actually contain π ?

(3 marks)

Since $n \geq 30$ by (CLT) assume $\hat{\pi}$ is Normally dist.

$$\text{M. Error} = 0.29 - 0.26 = 0.03$$

$$\boxed{Z \times \sqrt{\frac{0.26(1-0.26)}{400}} = 0.03} \checkmark \text{ (sets up equation)}$$

$$Z = 1.36788$$

$$\text{But } P(-1.36788 \leq Z \leq 1.36788) = 0.82865 \checkmark \text{ (Confidence Interval)}$$

Hence level of confidence is 82.9%.

ie ≈ 41 of the 50 intervals contain π .

\checkmark (Number of students)

End of questions

